Topic 11:

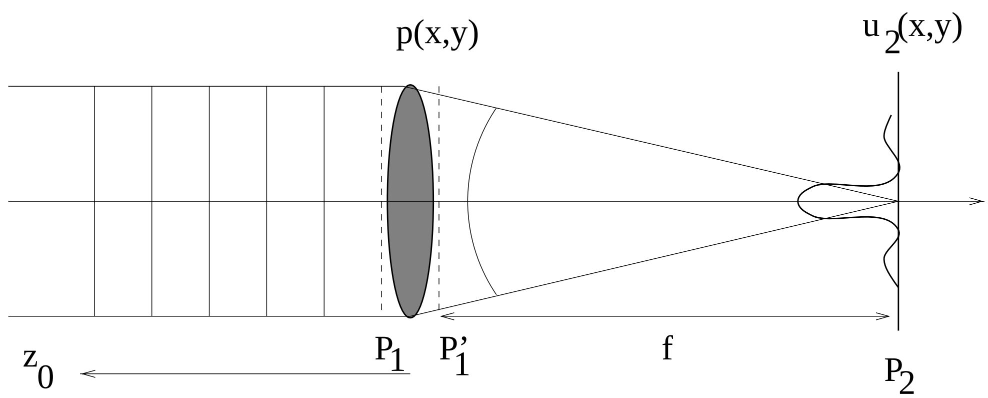
Optical Processing

Aim: These two lectures cover basic optical processing using the 4-f optical system with amplitude filters, phase filters, Fourier holograms and as a joint transform correlator. Finally the practicality of these systems is considered.

Contents:

Fourier Properties of a Lens

The Amplitude PSF of a lens is just the scaled Fourier Transform of its Pupil Function,



Then the amplitude in P_2 (including the quadratic phase factor) becomes,

$$\begin{aligned} u_2(x,y) &= B_0 \exp\left(\imath \frac{\kappa}{2f}(x^2+y^2)\right) \\ & \iint p(s,t) \exp\left(-\imath \frac{\kappa}{f}(sx+ty)\right) \mathrm{d}s\mathrm{d}t \end{aligned}$$

If we define

$$P(u,v) = \iint p(x,y) \exp\left(-\imath 2\pi(ux+vy)\right) \mathrm{d}x\mathrm{d}y$$

Then

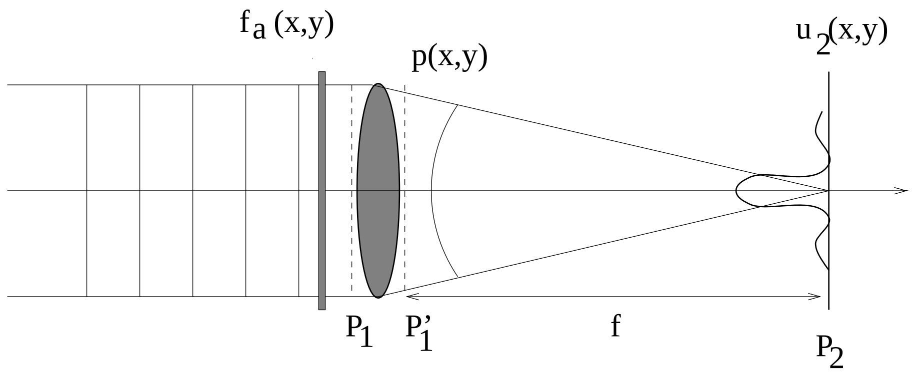
$$u_2(x,y) = B_0 \exp\left(\imath \frac{\kappa}{2f}(x^2+y^2)\right) P\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

Note: in units,

$$\begin{aligned} x,y &\rightarrow \text{Units of length, mm} \\ u,v &\rightarrow \text{Units of Spatial Freq, } \mathrm{mm}^{-1} \end{aligned}$$

Fourier Transform of Slide

Place slide of *Amplitude Transmittance* $f_a(x,y)$ close to lens



if $f_a(x,y)$ is **smaller** than the lens, $f_a(x,y)$ become *effective* pupil function,

$$\begin{aligned} u_2(x,y) &= B_0 \exp\left(\imath \frac{\kappa}{2f}(x^2+y^2)\right) \\ &\iint f_a(s,t) \exp\left(-\imath \frac{\kappa}{f}(sx+ty)\right) \mathrm{d}s\mathrm{d}t \end{aligned}$$

or more simply,

$$u_2(x,y) = B_0 \exp\left(\imath \frac{\kappa}{2f}(x^2+y^2)\right) F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

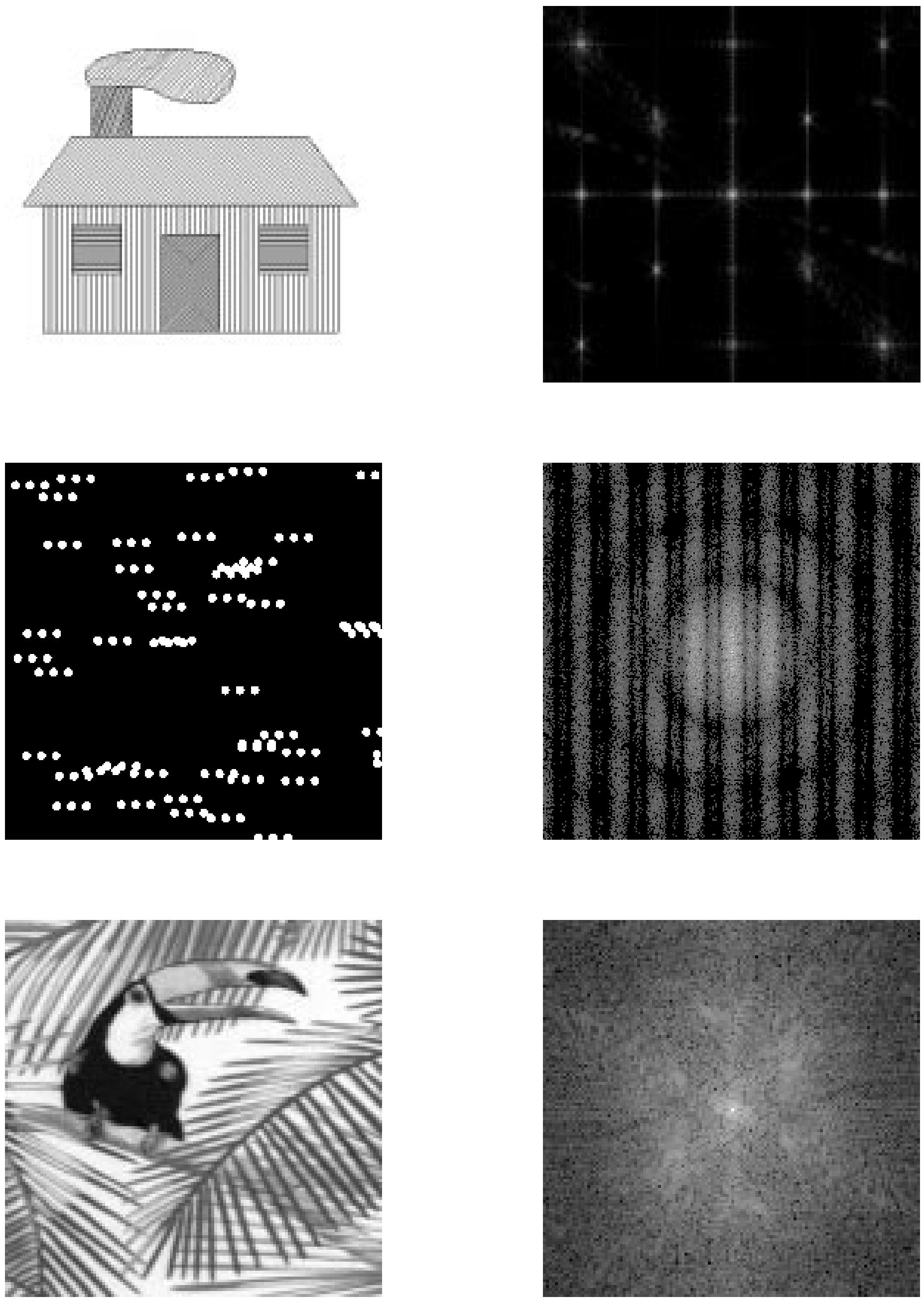
so in P_2 we get the scaled Fourier Transform of $f_a(x,y)$ plus a quadratic phase term.

The intensity in P_2 is then just

$$g(x,y) = B_0^2 \left| F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)\right|^2$$

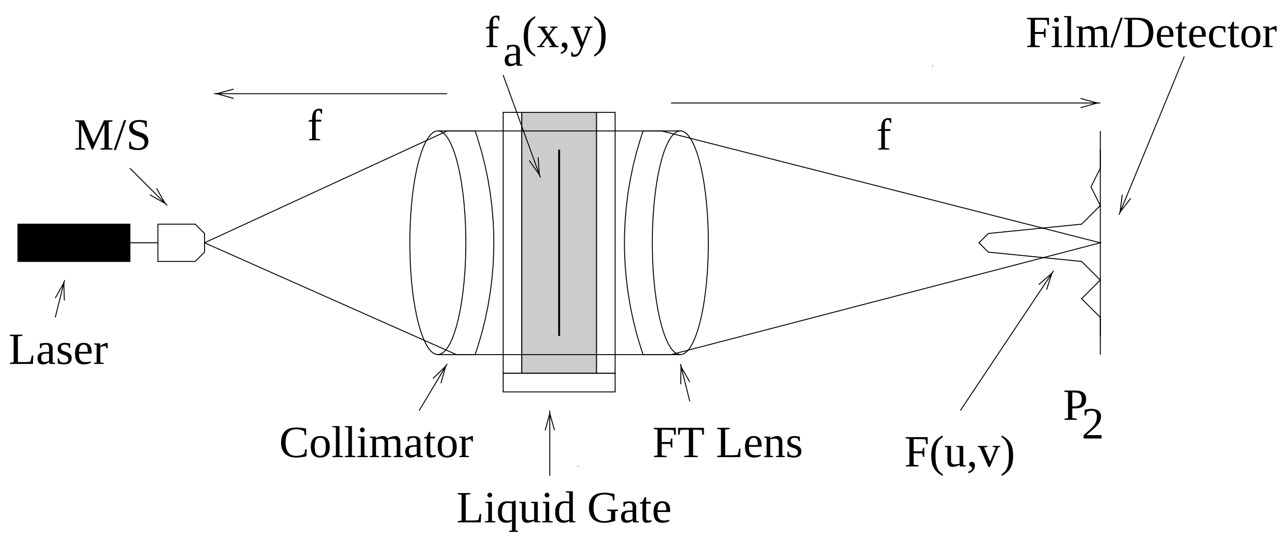
which is the Power Spectrum of $f_a(x,y)$

Fourier Transform Examples



Practical System

Typical practical system is



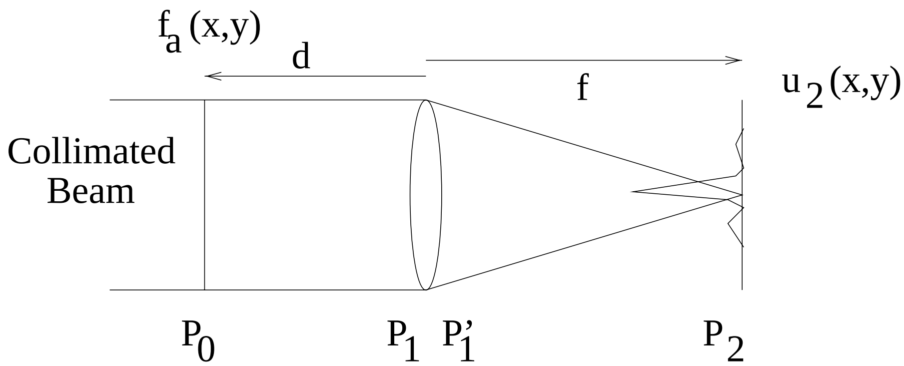
Focal length of lenses depends on expected frequency range, eg:

Maximum spatial frequency in $f_a(x,y)$:	100mm^{-1}
Size of Fourier plane:	$\pm 10\text{mm}$
Wavelength:	633nm
Focal Length FT Lens: 160mm	

Note: Usually need Liquid Gate to remove phase effect of gelatine to get good results

General Case

Move the object plane a distance z from the lens,



Use Fresnel diffraction to
1: Propagate from $P_0 \to P_1$
2: Lens adds a phase factor in P_1'
3: Propagate from $P_1' \to P_2$

If we can ignore the finite aperture of the lens, "it-can-be-shown" (see tutorial) that in P_2 we get

$$\begin{aligned} u_2(x,y) &= \exp\left(\imath\frac{\kappa}{2f}\left(1-\frac{z}{f}\right)(x^2+y^2)\right) \\ & \iint f_a(s,t)\exp\left(-\imath\frac{\kappa}{f}(xs+yt)\right)\mathrm{d}s\,\mathrm{d}t \end{aligned}$$

so if we take the special case of $z = f$, then we get

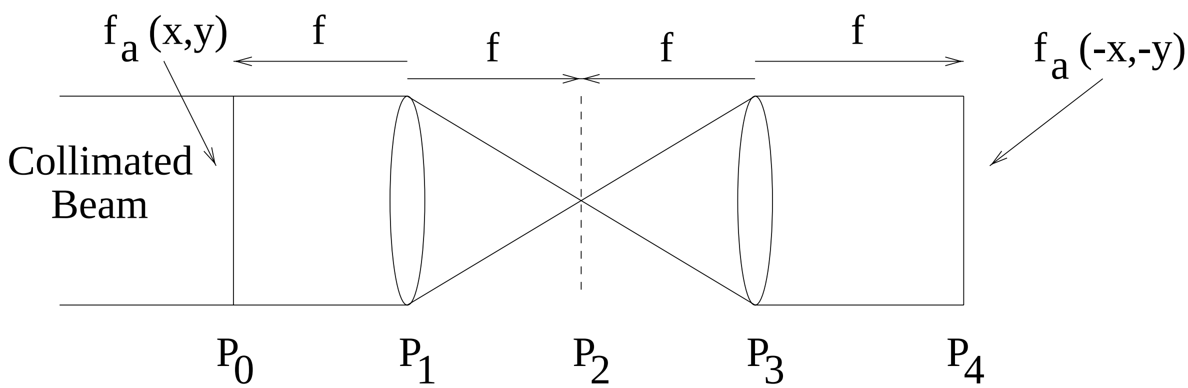
$$u_2(x,y) = \iint f_a(s,t)\exp\left(-\imath\frac{\kappa}{f}(xs+yt)\right)\mathrm{d}s\,\mathrm{d}t$$

so we get the exact Fourier transform, without any phase term, (external constants ignored)

$$u_2(x,y) = F\left(\frac{x}{\lambda f},\frac{y}{\lambda f}\right)$$

Optical Processing System

Put two lenses together to get "4-f Optical System".



Input light at P_0 is collimated, in P_2 we have

$$u_2(x,y) = F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

Second lens takes a second Scaled FT, so in P_4 we get

$$u_4(x,y) = f_a(-x,-y)$$

so a mirror image of the input.

The intensity measured in P_4 is then given by

$$g(x,y) = |f_a(-x,-y)|^2$$

but we usually rearrange the coordinates in P_4 so that we have

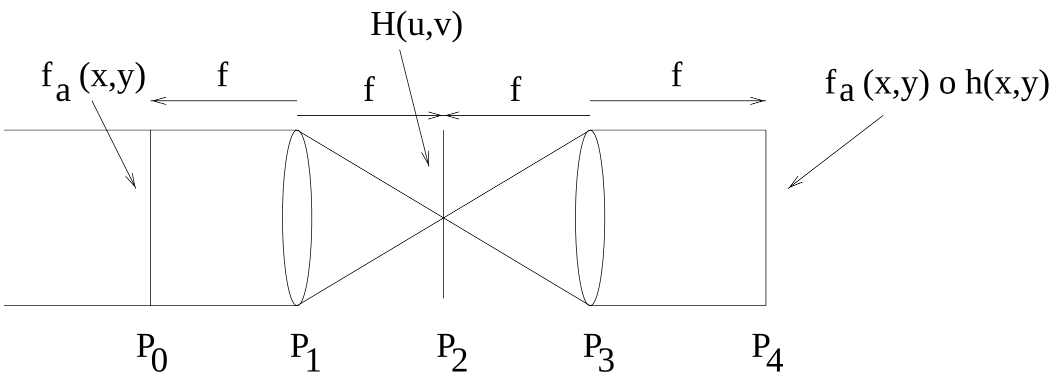
$$g(x,y) = |f_a(x,y)|^2$$

Note this assumes that the PSF is **small**, so valid for

Small Objects & Large Lenses

Convolution Filtering

In plane P_2 we have for Fourier Transform, so we can add a “filter”.



for and input of $f_a(x,y)$ in plane P_2 we have

$$F(u,v) \quad u = \frac{x}{\lambda f} \quad v = \frac{y}{\lambda f}$$

Apply a filter (slide) in P_2, modify the distribution to

$$F(u,v)\,H(u,v)$$

so output plane P_4 is Convolution, giving

$$u_4(x,y) = f_a(x,y) \odot h(x,y)$$

where

$$h(x,y) = \iint F(u,v)\exp(-\imath 2\pi(ux+vy))\,\mathrm{d}u\,\mathrm{d}v$$

so the **intensity** in P_4 is

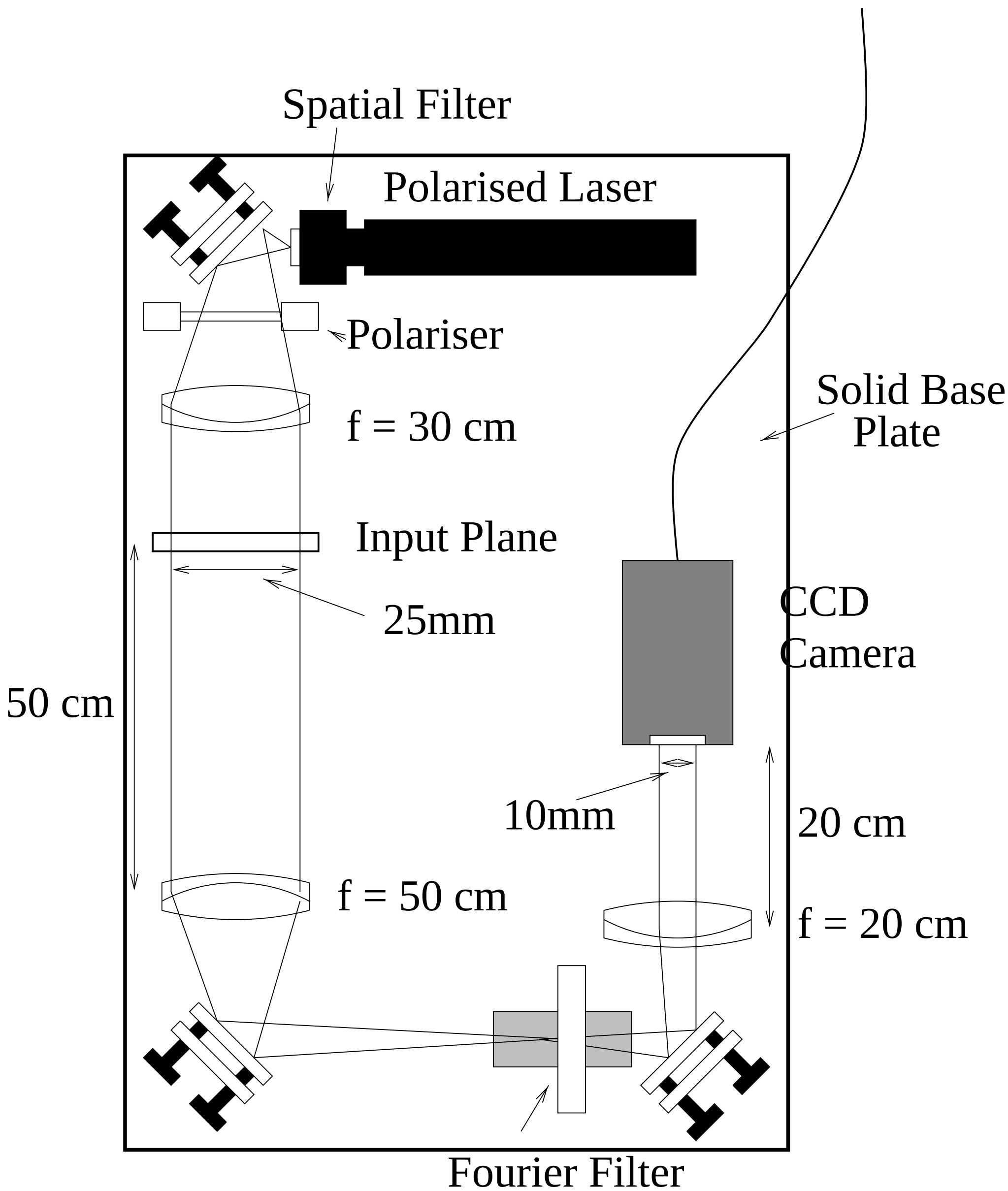
$$g(x,y) = |\, f_a(x,y) \odot h(x,y)|^2$$

So by changing $H(u,v)$ we can apply different types of filters, which are convolved with $f_a(x,y)$.

This system is the basis of Optical Image Processing.

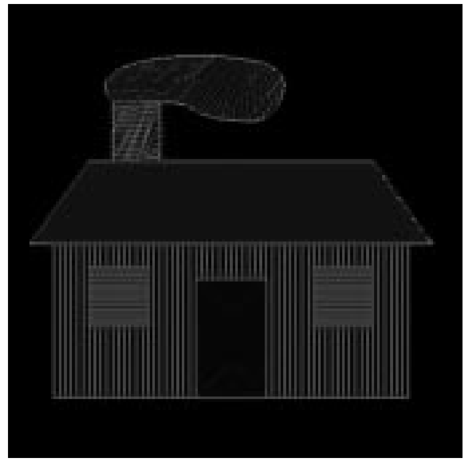
Practical System

To make Fourier plane of sensible size you need long focal length lenses, so typically have to “fold” system.



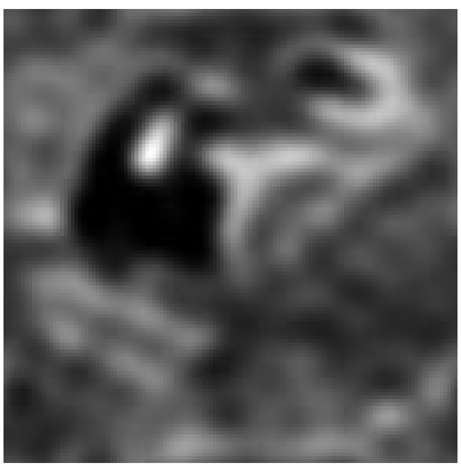
Need not have all lenses the *same* focal length, this system (as set-up in Optics Lab), has a magnification of $2/5$ to fit CCD array camera.

Filtering Examples



Low pass Filtered

High pass Filtered



Low pass Filtered

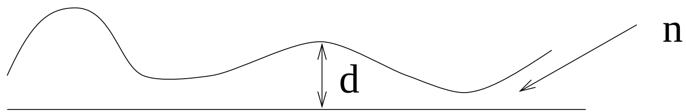
High pass Filtered

This is a digital simulation with some enhancement to show details.

See Hecht page 268 for examples.

Phase Objects

Phase object is a transparent object with structure associated with thickness variation.



Amplitude transmission of the object

$$f_a(x,y) = \exp\left(\imath\phi(x,y)\right)$$

where $\phi(x,y)$ is the Optical Path Difference, so:

$$\phi(x,y) = \frac{2\pi n d(x,y)}{\lambda}$$

where n is the refractive index.

In imaged in either Coherent or incoherent light, see

$$g(x,y) = |\,f_a(x,y)|^2 = 1$$

so we don't see any structure.

This problem occurs in:

1): Biological cells, $\approx 98\%$ water
2): Photo-resist on glass, (VLSI, and holograms) Common prob-
3): Finger prints on glass.

lem is microscopy.

Thin Phase Approximation

Phase is periodic of period 2π, can write

$$f_a(x,y) = \exp(\imath\phi_0)\exp(\imath\phi(x,y))$$

where we have that

$$-\pi < \phi(x,y) \leq \pi$$

and ϕ_0 take no part in the imaging process.

Take the "Weak-Phase" approximation, expand $f_a(x,y)$ to get

$$f_a(x,y) \approx 1 + \imath\phi(x,y) - \frac{\phi^2(x,y)}{2}$$

so if $|\Phi| \ll 1$ take the approximately for first order, then

$$f_a(x,y) \approx 1 + \imath\Phi(x,y)$$

This is valid for many practical cases, for example biological cells in water.

Take the Fourier transform of this, typically optically, in a 4-f optical system, the Fourier Transform

$$F(u,v) = \delta(u,v) + \imath\Phi(u,v)$$

where

$$\Phi(u,v) = \mathcal{F}\,\{\phi(x,y)\}$$

Add Filter in Fourier plane to make phase distribution visible as an Intensity.

Dark Field

Apply filter of

$$\begin{aligned} H(u,v) &= 0 \quad u^2+v^2=0 \\ &= 1 \quad \text{else} \end{aligned}$$

(Filter is a “black spot”)

After filter we get

$$\begin{aligned} F(u,v)H(u,v) &= 0 \quad u^2+v^2=0 \\ &= \imath\Phi(u,v) \quad \text{else} \end{aligned}$$

so in P_4 after second Fourier Transform we get

$$u_4(x,y) = \imath\phi(x,y)$$

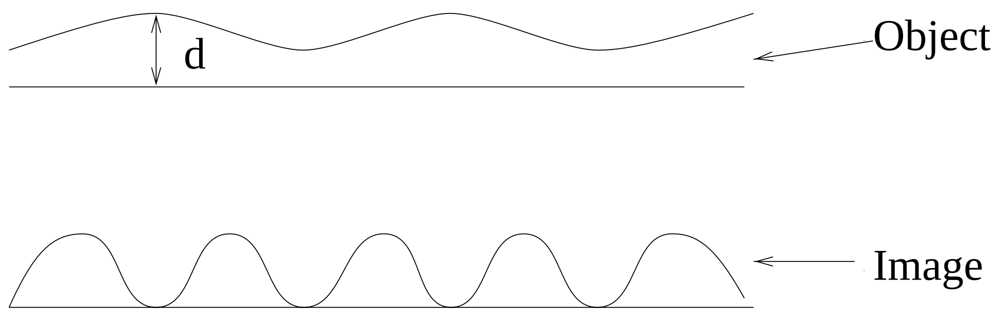
so intensity in output is

$$g(x,y) = |\phi(x,y)|^2$$

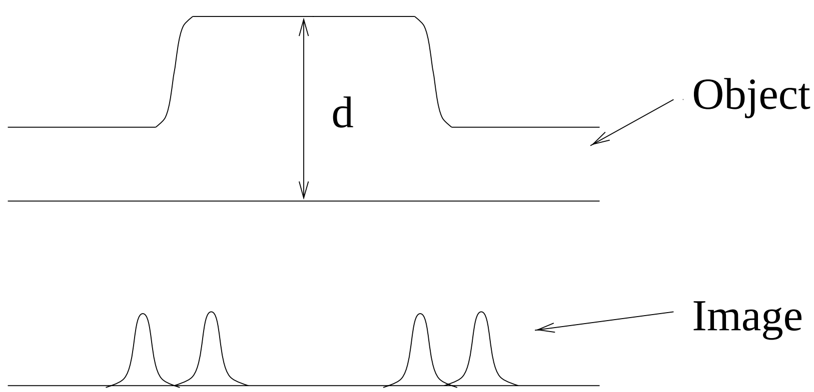
so the phase variation become visible.

Aside: Called “Dark Field” if no object, no light through system

Practical Problem: detect $|\phi(x,y)|^2$ so we get apparent frequency doubling, eg for $\cos$ variation,



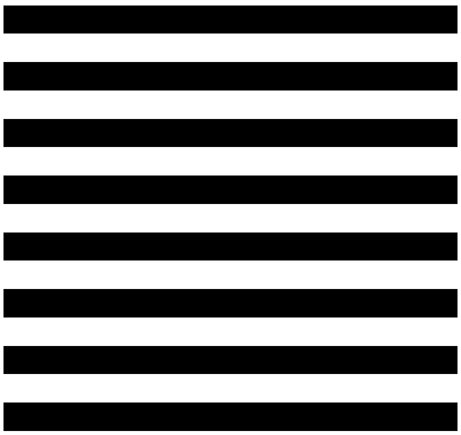
This also occurs at all spatial frequencies, which results in double edges.

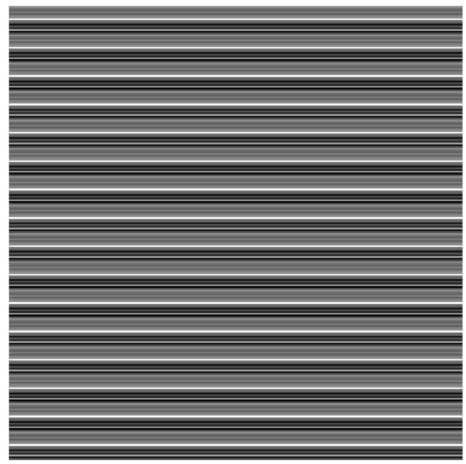


which makes images difficult to interpret

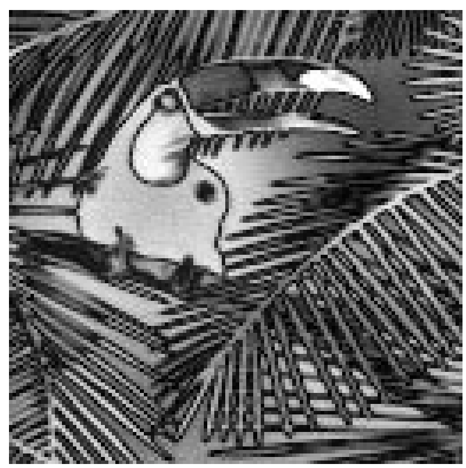
Dark Field Examples

Digital simulation with a maximum phase modulation of $\phi \approx 0.5$


Phase Grating


Dark Field Reconstruction


Phase Toucan


Dark Field Reconstruction

Both images show edge doubling. They are actually differentials of the phase.

Phase Contrast Filtering

Zernike 1940, (Nobel Prize 1953)

In Fourier space we have

$$F(u,v) = \delta(u,v) + \imath\Phi(u,v)$$

apply a filter of

$$\begin{aligned} H(u,v) &= \exp(\imath\pi/2) \quad u^2+v^2=0 \\ &= 1 \quad \text{else} \end{aligned}$$

Filter is a "dot" of $\lambda/4$ optical path length.

So after filter we get

$$F(u,v)H(u,v) = \imath\,(\delta(x,y) + \Phi(u,v))$$

now in P_4 after the second Fourier Transform,

$$u_4(x,y) = \imath\,(1+\phi(x,y))$$

so we see intensity

$$g(x,y) = |1+\phi(x,y)|^2 = 1+2\phi(x,y)+\phi^2(x,y)$$

but is ϕ is small, then

$$g(x,y) \approx 1+2\phi(x,y)$$

to the intensity of the output is **linear** phase

Phase filter fairly difficult to make, original was an oil-drop" now made by film evaporation.

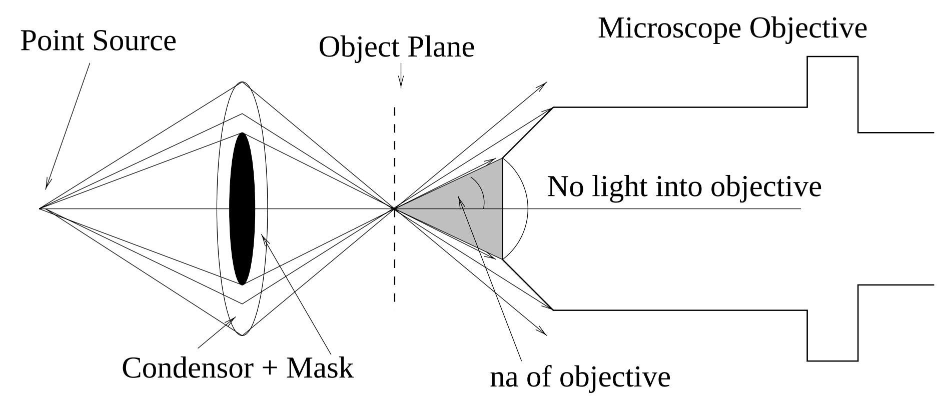
See Guenther Page 414 figure 10B-15 (b) & (c) for good example

Practical Uses

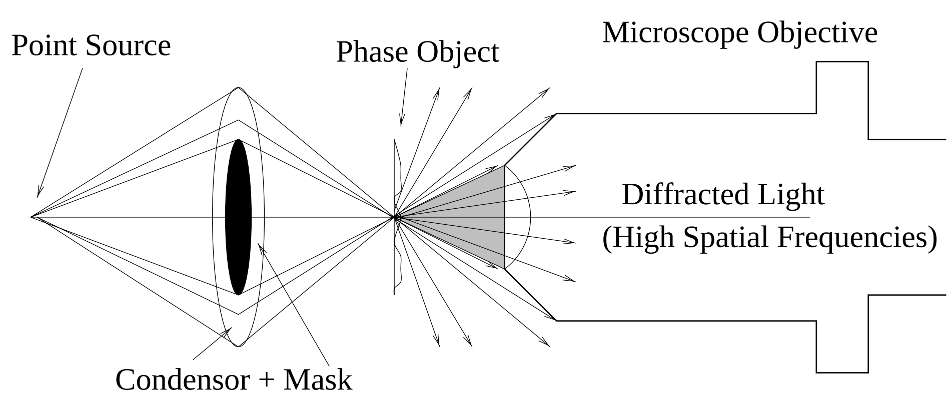
Both “Dark Field” and “Phase Contrast” frequently used in microscopy.

In microscope filters **not** placed in Fourier plane, but designed into Condenser and Microscope objective.

Dark field microscope system, no object to no light to image.



Add phase object, diffracted light to image.



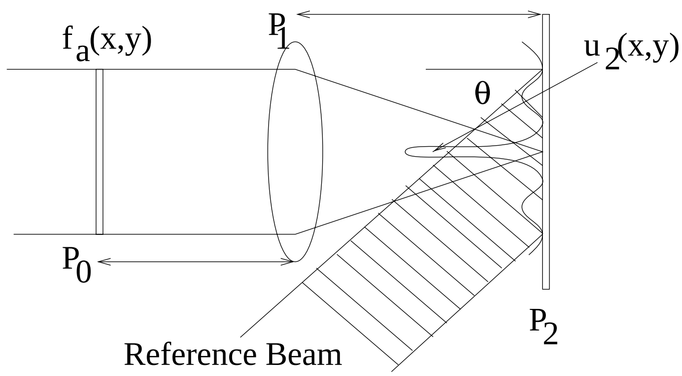
Systems look different from 4-f but same mathematics

Range of other filtering techniques used, for example colour filtering, polarisation interference etc.

Fourier Holograms

Filters are either Amplitude or Phase, but not arbitrary complex. Look how holography can help.

Consider system:



In P_2 we have

$$u_2(x,y) = F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)$$

so Scaled Fourier Transforms of input transparency.

Add an off-axis reference beam at angle θ. Intensity in P_2 is

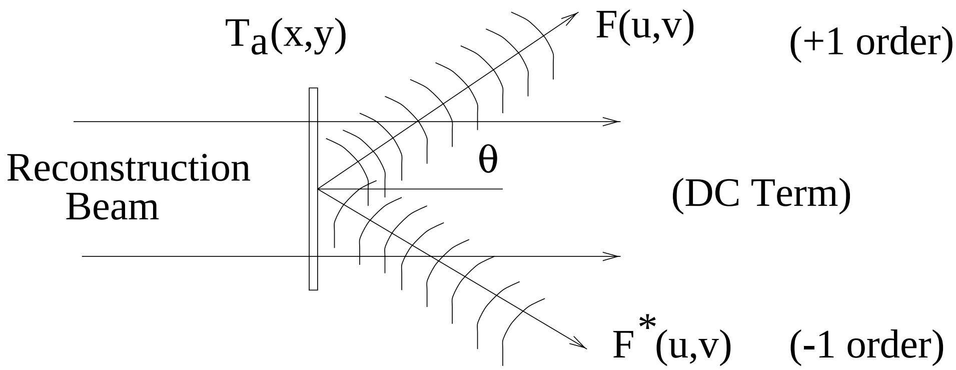
$$|r\exp(\imath\kappa x\sin\theta) + u_2(x,y)|^2$$
$$r^2 + |u_2(x,y)|^2 + 2r|u_2(x,y)|\cos(\kappa x\sin\theta - \Phi)$$

where $u_2(x,y) = |u_2(x,y)|\exp(\imath\Phi)$.

So hologram then encodes $u_2(x,y)$, so $F(u,v)$

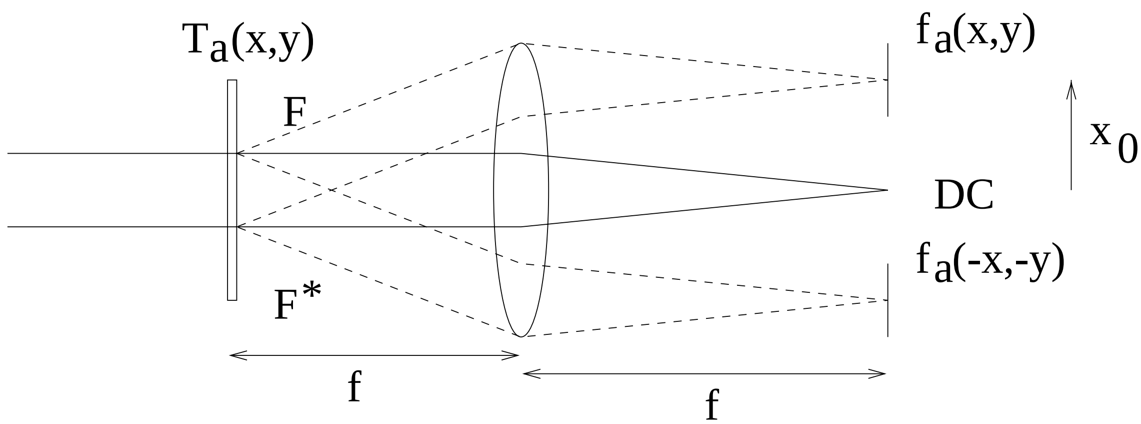
Reconstruction

Plane Beam: (at angle $\theta = 0$)



Three terms, these being DC and ± 1 order in $\pm\theta$ direction.

Take Fourier Transform:



Three terms get Fourier Transformed to give images.

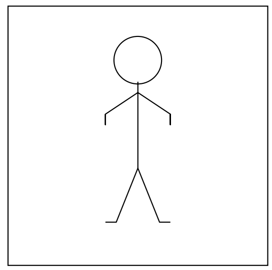
From geometry we have that

$$x_0 = f \tan\theta = \approx f\theta$$

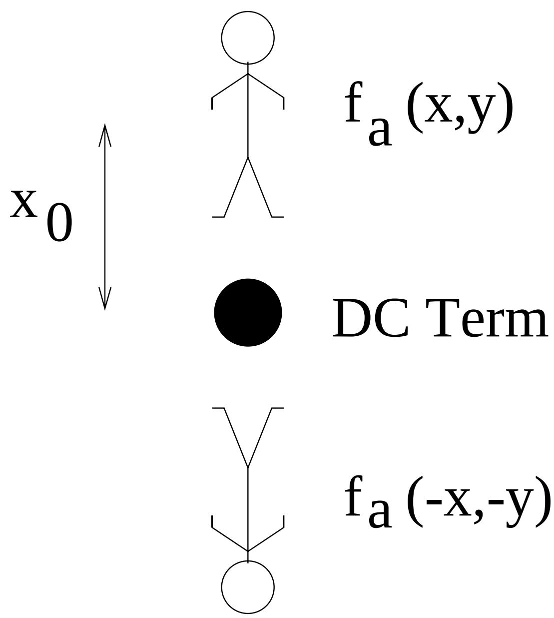
so in the output we get **three** terms

$$\delta(x,y) + f_a(x,y) \odot \delta(x - x_0) + f_a(-x,-y) \odot \delta(x + x_0)$$

So for input of $f_a(x,y)$,



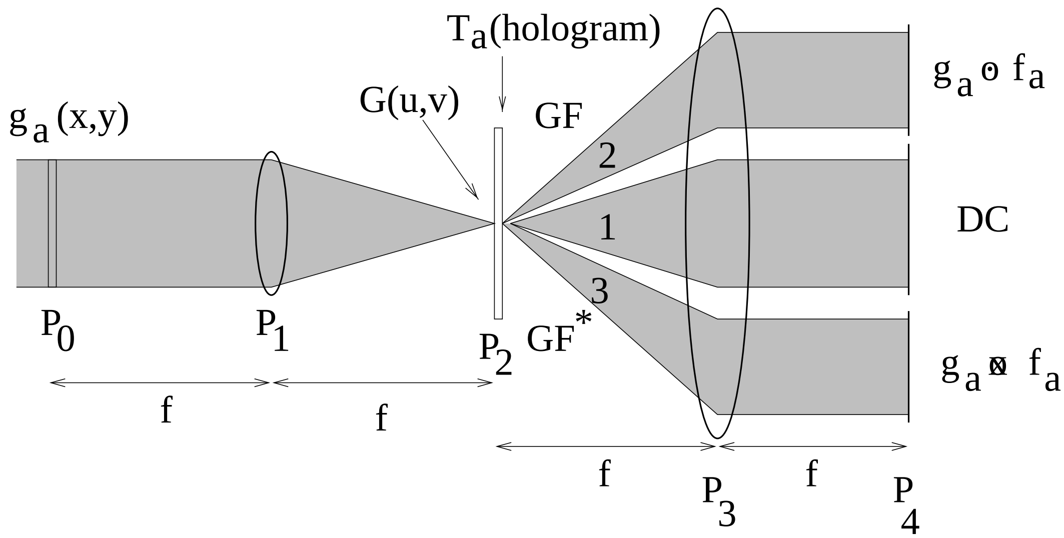
We get an output of



This is not really useful in itself, but use filter in Fourier plane of 4-F system.

Vander Lugt Correlator (1966)

Place hologram containing hologram of $f_a(x,y)$ in Fourier plane of 4-F system.



For input of $g_a(x,y)$ before P_2, we get $G(u,v)$

After P_2, THREE terms:

1. DC Term (Mixed term, not useful)
2. $G(u,v)F(u,v)$ in direction θ
3. $G(u,v)F^*(u,v)$ in direction $-\theta$

Fourier Transformed by Lens, to give in P_4

1. Mixed term on-axis (not used)
2. $g_a(x,y) \odot f_a(x,y)$ located about x_0
3. $g_a(x,y) \otimes f_a(x,y)$ located about $-x_0$

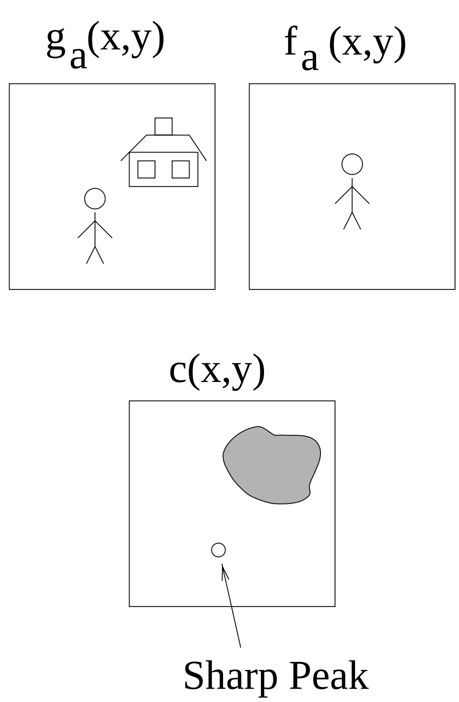
So if θ large enough, THREE terms are separated.

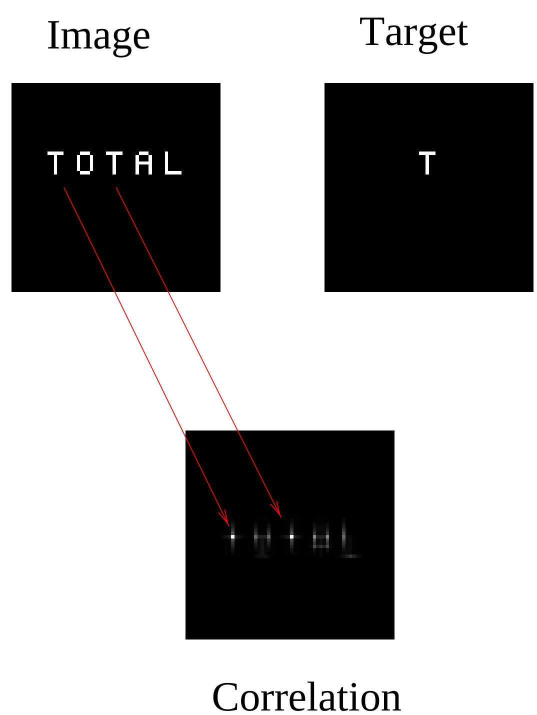
Note we actually detect,

$$|g_a(x,y) \odot f_a(x,y)|^2$$
$$|g_a(x,y) \otimes f_a(x,y)|^2$$

This is not a problem since $g_a(x,y)$ and $f_a(x,y)$ are Real and Positive.

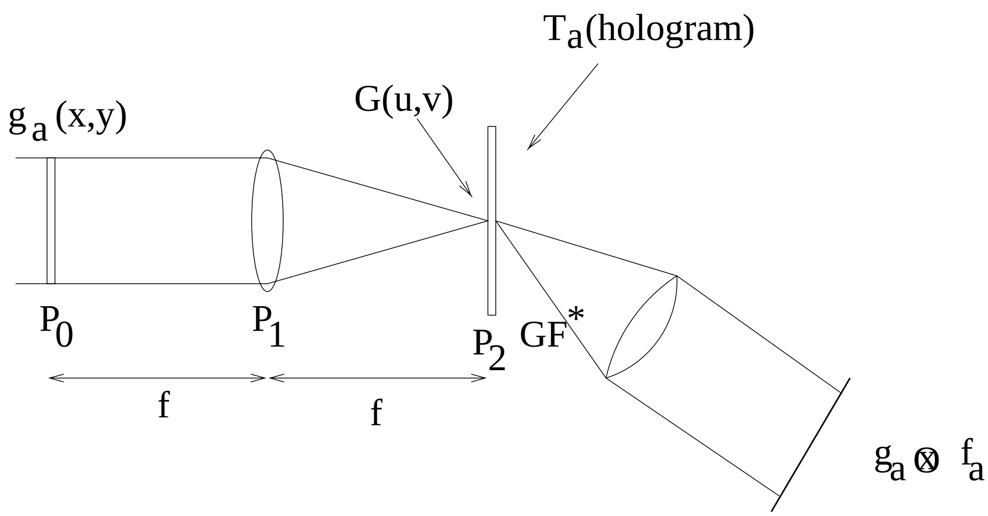
General method of correlation between two image scenes, so the basic for real time optical tracking





- Height of correlation peak gives “Closeness of Match”
- Location of peak gives “Location of Target”

Practical System



Range of Problems:

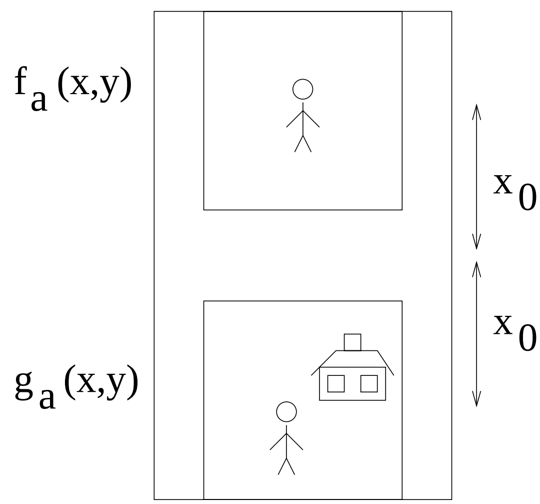
1. **Dynamic range of FT:** Difficult to make hologram to encode all of $F(u,v)$. Edge enhancement effects.
2. **Target and Scene differences:** 2% scale or 2° rotation results in 50% drop in correlation.
3. **Single Target:** One target of hologram, difficult to change.
4. **Not Real Time:** photographic negative input, (Solved by SLM, next lecture).
5. **Alignment Problems:** Location of hologram is critical.

Despite this can be made to work, (hand-held version made).

Joint Transform Correlator

Variant on the Vander-Lugt, with potential for real time input and target.

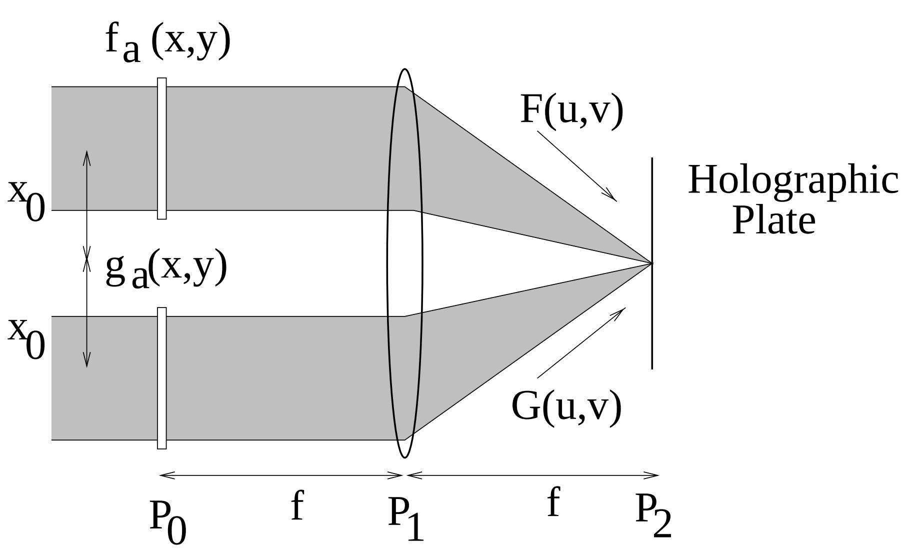
For target $f_a(x,y)$ and Scene $g_a(x,y)$,arrange as:



which we can write at

$$f_a(x,y) \odot \delta(x - x_0) + g_a(x,y) \odot \delta(x + x_o)$$

Fourier Transform this (optically), to get



In plane P_2 we get amplitude,

$$F(u,v)\exp(-\imath 2\pi x_0 u) + G(u,v)\exp(\imath 2\pi x_0 u)$$

Record this as an **Intensity** (on holographic plate), to get:

$$= |F|^2 + |G|^2 + FG^* \exp(-\imath 4\pi x_0 u) + F^* G \exp(\imath 4\pi x_0 u)$$

which we can write as:

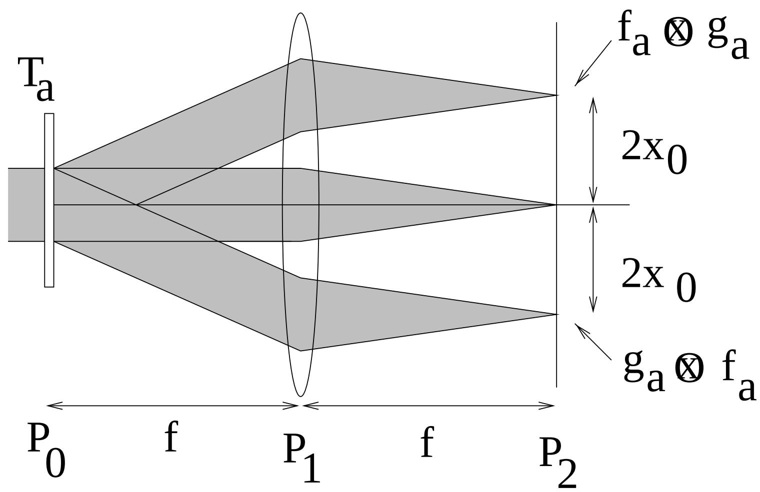
$$|F|^2 + |G|^2 + 2|FG^*|\cos(4\pi x_0 u - \Phi)$$

where $FG^* = |FG^*|\exp(\imath\Phi)$.

This is actually a **hologram** that results from the interference between $F(u,v)$ and $G(u,v)$.

Note: this is practically difficult since both $F(u,v)$ and $G(u,v)$ are Fourier Transforms, which have a dynamic range much greater than the holographic film.

Take hologram and illuminate with collimated beam, and take optical Fourier transform. We get THREE terms



1. $f_a \otimes f_a + g_a \otimes g_a$. On-axis mixed term, (not useful)
2. $f_a \otimes g_a \odot \delta(x - 2x_0)$ cross-correlation located about $2x_0$
3. $g_a \otimes f_a \odot \delta(x + 2x_0)$ cross-correlation located about $-2x_0$

If x_0 is large enough then we get the three terms separated, and can get $f_a \otimes g_a$.

Again we actually detect $|f_a(x,y) \otimes g_a(x,y)|^2$.

Problems:

1. **Dynamic Range of Fourier Hologram:** Same as in Vander Lugt case.
2. **Two stage process:** Need a hologram for each recognition. (this look worse).
3. **Light Efficiency:** very poor use of light, hologram is very inefficient.

To make system "real time" need to record "real-time" hologram, which will be discussed in the next lecture.

Aside: Can be simplified by recording "hologram" on TV camera and taking the second Fourier Transform digitally.