Topic 2: Imaging Process

Aim

This lecture covers the parts of optical image formation required for this course.

Contents:

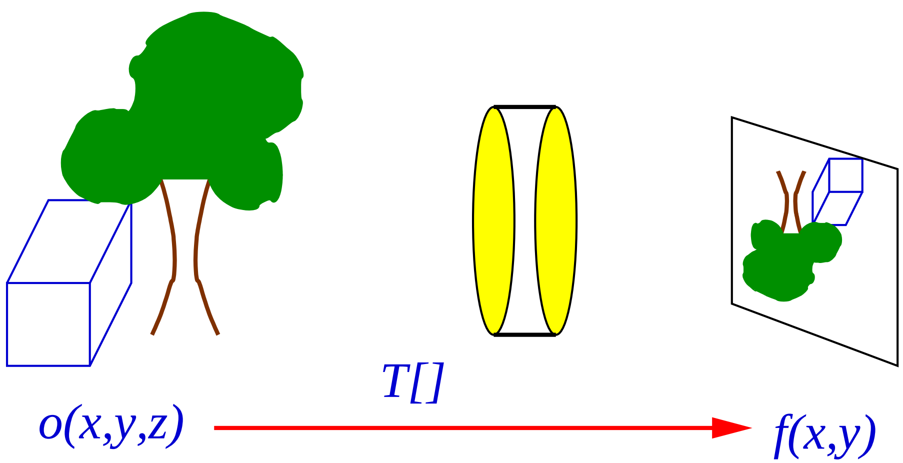
1. Imaging as a Linear Process
2. PSF of ideal system
3. OTF of ideal system
4. Validity of assumptions

Important material in *"Fourier Transform" (What you need to know).*

Imaging as a Linear Mapping

Before starting must understand the image formation process.

Generally 3-Dimensional object imaged to 2-Dimensional plane.



which we can write as:

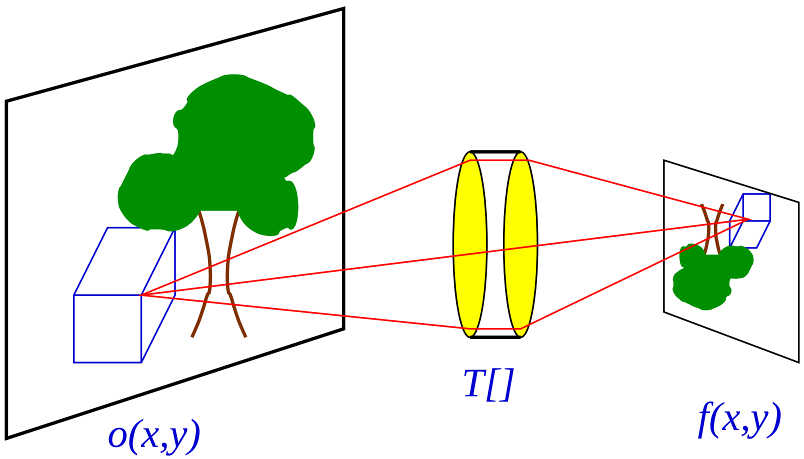
$$f(x,y) = T[o(x,y,z)]$$

where $f(x,y)$ is the detected intensity.

Imaging as a Linear Mapping I

Initially assume object is 2-Dimensional,

$$f(x,y) = T[o(x,y)]$$



This will result in problem, which we consider later

Linearity

If we have **two** objects $o_1(x,y)$ & $o_2(x,y)$ of brightness α and β. Detected image:

$$f(x,y) = T\left[\alpha o_1(x,y) + \beta o_2(x,y)\right]$$

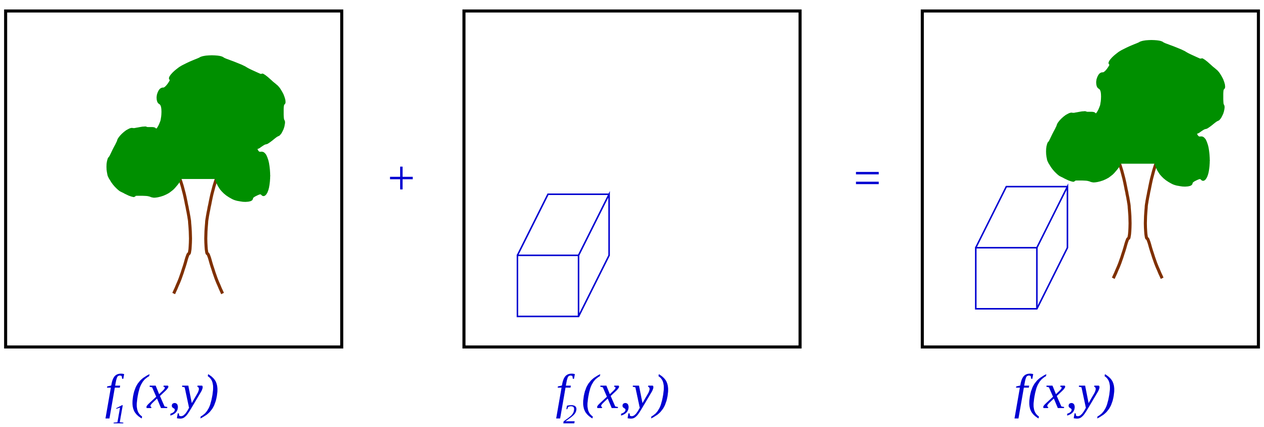
but if T is *linear* we get,

$$f(x,y) = \alpha T[o_1(x,y)] + \beta T[o_2(x,y)] = \alpha f_1(x,y) + \beta f_2(x,y)$$

where

$$f_1(x,y) = T[o_1(x,y)] \quad \text{and} \quad f_2(x,y) = T[o_2(x,y)]$$

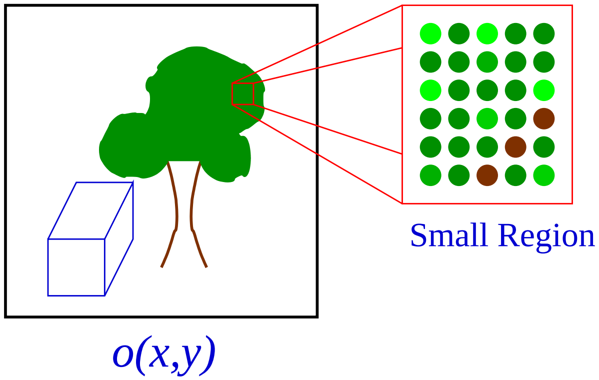
which are the images of the two objects.



Problem with overlapping object. Problem due to assumption that object is two-dimensional.

Image of Points

Consider the input scene $o(x,y)$ as a series of closely arranged "points"



we can consider *each* point as a separate object, and we can form the image of each *point* separately.

Consider each *point* as a two-dimensional δ-function. *Point* at position a,b is represented by

$$\delta(x-a, y-b)$$

Using shifting properties of δ-function, we can write

$$o(x,y) = \iint o(s,t)\delta(x-s, y-t)\,\mathrm{d}s\mathrm{d}t$$

(Convolution with δ-function is a null operation).

Image of Points I

We have that

$$f(x,y) = T[o(x,y)]$$

So substitution for $o(x,y)$ gives:

$$f(x,y) = T\left[\iint o(s,t)\delta(x-s,y-t)\mathrm{d}s\mathrm{d}t\right]$$

If $T[]$ is *linear*, we can re-arrange to get:

$$f(x,y) = \iint o(s,t)T[\delta(x-s,y-t)]\mathrm{d}s\mathrm{d}t$$

We can then write

$$T[\delta(x-s,y-t)] = h(x,s,y,t)$$

Image of Points II

Where we can interpret as

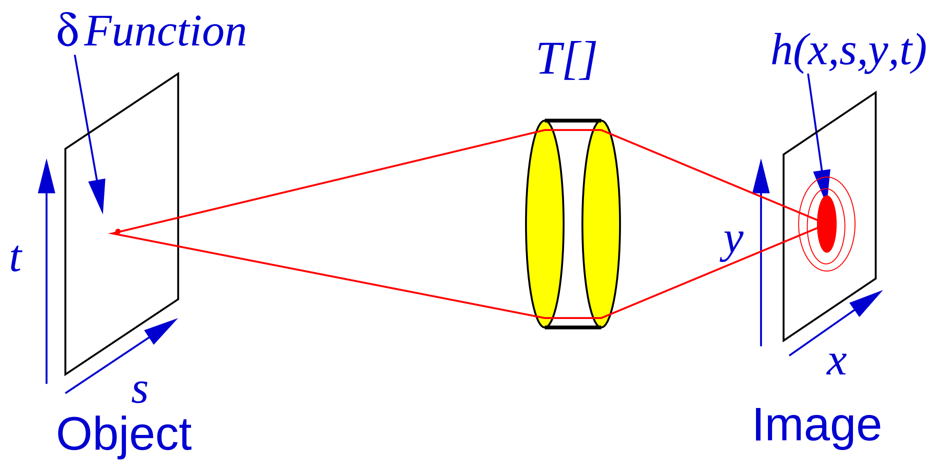


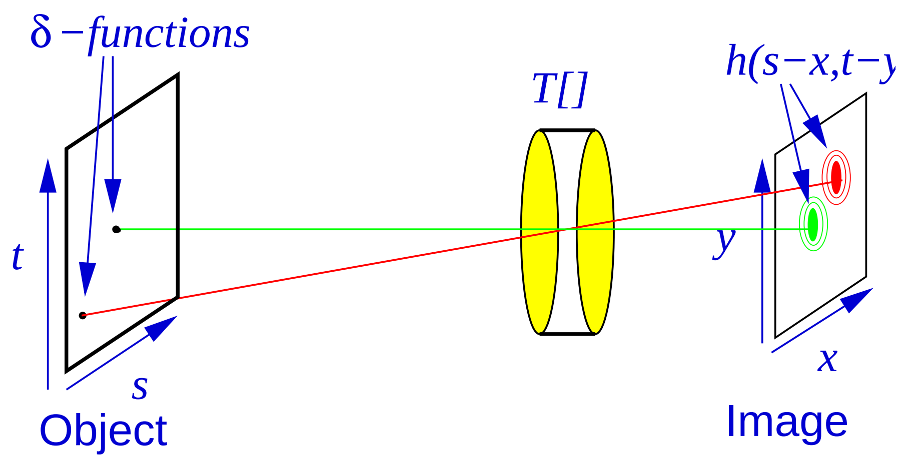
Image of a δ-function at position s, t, known as the *Point Spread Function*.

So the detected image is made up of the addition of a whole series of *Point Spread Functions*

Space Invariant

Assume system is *Space Invariant*, so shape of $h(x,s,y,t)$ does **not** depend on s,t,

$$h(x,s,y,t) = h(x-s,y-t)$$



so we have that

$$f(x,y) = \iint o(s,t)\,h(x-s,y-t)\,\mathrm{d}s\mathrm{d}t$$

which is just the *Convolution* integral in two dimensions, which we will write as

$$f(x,y) = o(x,y) \odot h(x,y)$$

Aside: This formulation assume unit magnification. (For other system mathematics are the same)

In Fourier Domain

If in *Real Space* we have,

$$f(x,y) = o(x,y) \odot h(x,y)$$

then from the *Convolution Theorem* we have that

$$F(u,v) = O(u,v)\,H(u,v)$$

where

$$\begin{aligned} F(u,v) &= \mathcal{F}\{f(x,y)\} && \text{FT of the Image} \\ O(u,v) &= \mathcal{F}\{o(x,y)\} && \text{FT of the Object} \\ H(u,v) &= \mathcal{F}\{h(x,y)\} && \text{Filter Function} \end{aligned}$$

This is the main reason that Fourier Theory is used imaging.

In Fourier Domain I

For an imaging system, we have

$$h(x,y) \rightarrow \text{ Point Spread Function}$$

being the *image of a star*.

Thus $H(u,v)$ acts like a *Fourier Space Filter* or *Transfer Function* from object to image,

$$H(u,v) \rightarrow \text{ Optical Transfer Function}$$

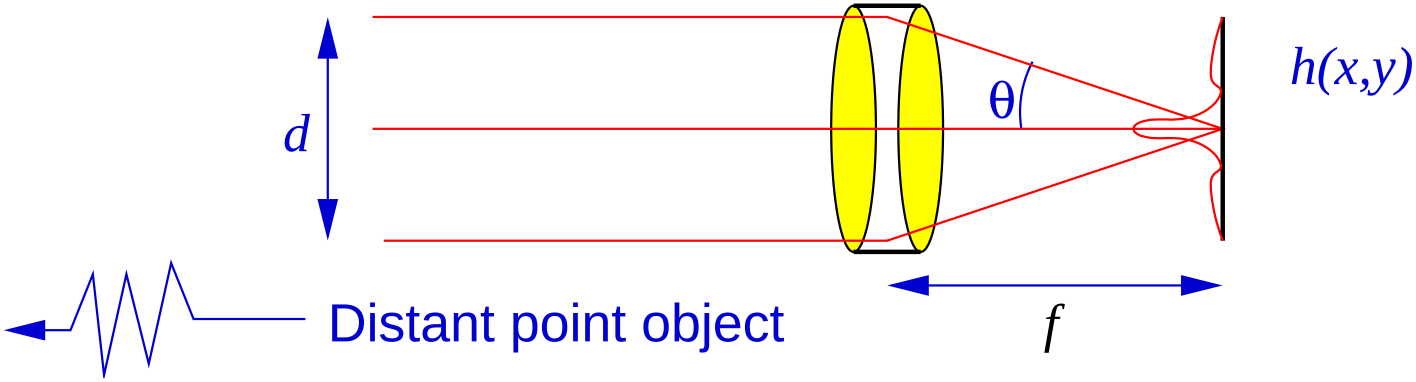
If we know *either* $h(x,y)$ *or* $H(u,v)$ we are able to fully characterise the system by either, since

$$h(x,y) \Leftrightarrow H(u,v)$$

This gives us a *single function* to characterise the imaging properties.

Properties of Imaging System

For an ideal lens imaging a distant object,



"It-can-be-shown" (not part of course) that

$$h(x,y) = \left[\frac{J_1(\alpha\kappa r)}{\alpha\kappa r}\right]^2$$

where

$$\alpha = \sin\theta \quad \kappa = \frac{2\pi}{\lambda} \quad r = \sqrt{x^2 + y^2}$$

and $J_1()$ is the First order Bessel function.

Properties of Imaging System

Most optical system we can approximate

$$\alpha \approx \frac{d}{2f} = \frac{1}{2\mathrm{F_{No}}}$$

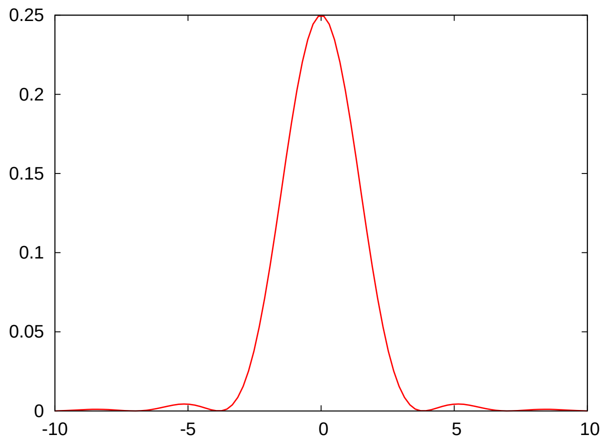
Where $\mathrm{F_{No}}$ is the F-number of the lens, which is defined as

$$\mathrm{F_{No}} = \frac{\text{Focal Length}}{\text{Diameter}}$$

So the PSF of an ideal lens is given by $\mathrm{F_{No}}$ and *Wavelength* of light only.

Shape of the PSF

The function $(\mathrm{J}_1(x)/x)^2$ has shape,



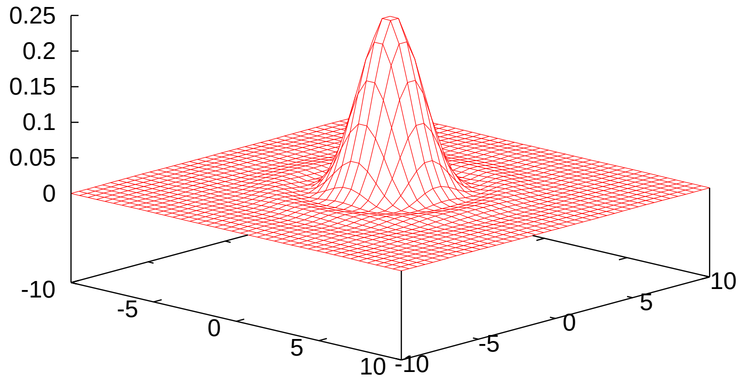
With the First Zero at

$$x_0 = 1.22\pi$$

being similar in shape to a $\sin^2()$.

Shape of the PSF I

To in two-dimensions we get



with the first zero beig as radius

$$r_0 = 1.22\lambda F_{No}$$

The PSF has 88% of the energy in the central peak and a series of decreating intensity rings.

Optical Transfer Function

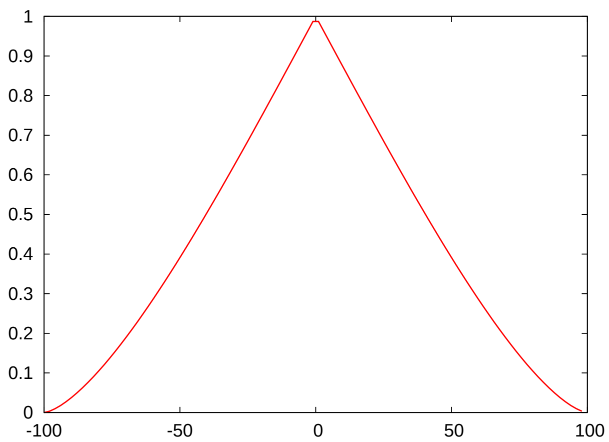
Again this can be calculated analytically, as *can-be-shown* to be given by:

$$H(u,v) = \frac{2}{\pi}\left[\cos^{-1}\left(\frac{w}{v_0}\right) - \frac{w}{v_0}\left(1-\left(\frac{w}{v_0}\right)^2\right)^{\frac{1}{2}}\right]$$

where $w = \sqrt{u^2+v^2}$ and

$$v_0 = \frac{d}{\lambda f} = \frac{1}{\lambda F_{\mathrm{No}}}$$

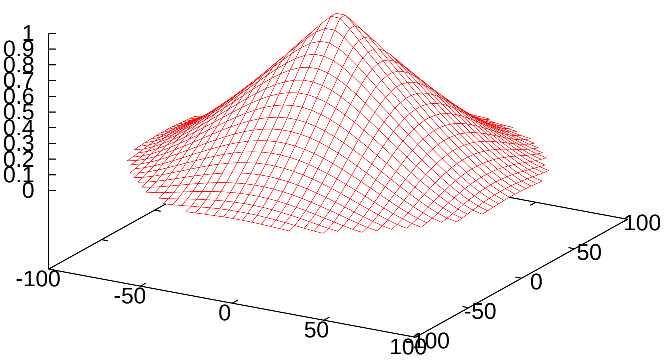
This function is tent shaped,



so spatial frequencies passed up to the limit of v_0, but **NOT** with equal amplitude.

Shape of OTF

The OTF is circularly symmetric, so shape is given by



The OTF acts as a *Fourier Filter*, so modifies the Fourier transform of the detected image.

Shape of OTF I

The OTF is a measure of the fidelity with which each spatial frequency (grating) is passed, but also

$$H(u,v) = 0 \quad \text{for } u^2 + v^2 > v_0^2$$

and we have that

$$F(u,v) = H(u,v)\, O(u,v)$$

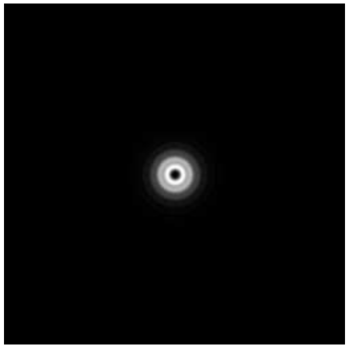
so detected image $f(x,y)$ has a Spatial Frequency limit set by the imaging system.

- Not all spatial frequencies passed with the same fidelity
- Spatial Frequency limit even for ideal system, so all imaging systems band-width limited.

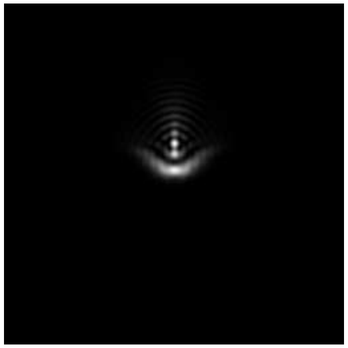
These properties allow us to represent an image in a computer, (see next Topic).

Non Ideal Lens

If the lens is not ideal, is has *aberration*, we can then calculate $h(x,y)$ digitally,



Defocus



Mixed “mess”

Original

Defocused

or we can detect the image of a distant point object (a star).

Non Ideal Lens I

Knowing $h(x,y)$ and hence $H(u,v)$ will allow us to digitally correct for aberrations, and (hopefully) reconstruct the ideal image.

This is covered in Setion 7, but...
Little more complex than that. . .

You can also measure $H(u,v)$, "too much optics" for this course.

Validity of Assumptions

To allow use of *Convolution Theorem* we have assumed

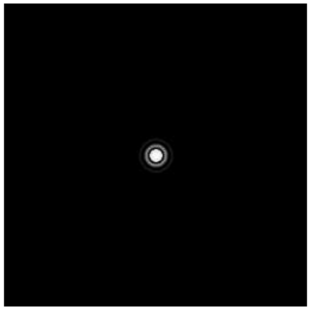
Linearity: Valid for most system, but problems with

1. 3-D scenes, where one object obscures the other
2. Photographic film and video frequently non-linear, (but we can allow for).
3. Saturation in sensors, particularly CCDs
4. Low light level (counting photons)

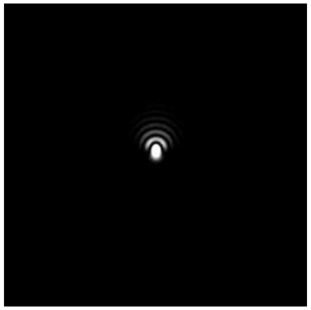
Validity of Assumptions

Space Invariance: Valid for most *Good* imaging systems, problems with

1. Large telescopes, aberrations of Coma.



Centre



Edge

2. Geometric distortions, (have to deal with separately)

If valid we get a simple *Fourier* model of imaging characterised by PSF or OTF.

If not image processing/restoration **very** much more difficult, (usually have to use trial-and-error)

Summary

In this section we have covered:

1. Basics of image formation in optical system.
2. Used the assumptions of linearity and space invariance for form a Fourier based model of imaging.
3. Considered the implications of this model for an ideal, perfect, lens.
4. Outlines the affect of aberrations and the resultant degraded imaging quality.
5. Outlined the validity of the underlying assumptions with reference to real imagin systems.